Identification of Steam Pressure Model for Once-through Steam Generator

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Abstract. This paper presents an input-output model of once-through steam generator, which describes the relationship between feed-water pressure and steam pressure. The identification process of mathematical model combines mechanism and data modeling; model structure is determined by physical mechanism, and unknown parameters of given model are estimated via total least squares and linear optimization. Using operation data of real system, the validity of proposed model and parameter estimation method is verified. Calculation results shows that the proposed model can well interpret and predict the state of real system.

Keywords: Once-through steam generator, Input-output model, Steam pressure, System identification, Total least squares, Linear optimization

1. INTRODUCTION

As the core equipment of marine nuclear power plant, once-through steam generator(OTSG) has been widely researched and applied at home and abroad, due to the advantages of generation of superheated steam, simple structure, small size, good static characteristics, good maneuverability and capacity to improve thermal efficiency of devices. Obviously, mathematical model of once-through steam generator is an important foundation for its application and research such as controller design, status monitoring and fault diagnosis. Therefore, modeling of once-through steam generator system is of great significance in theoretical analysis and engineering applications on it.

In recent decades, many researchers have studied models of once-through steam generator. The main method is to analyze heating section, evaporation section and superheat section of it based on global energy, mass and momentum balance respectively [1]. For example, Fu M Y et al. modeled once-through steam generator using lumped parameter method, and carried out static and dynamic analysis [2]; Liu D et al. thoroughly analyzed the flow and heat exchange laws of spiral tube once-through steam generator, and clarified the flow and heat transfer models of primary and secondary sides [3]; Chen X H established a one-dimensional dynamic mathematical model that can describe the working fluid characteristics of secondary circuit based on three major conservation laws that fluid obeys [4]. These documents analyze the fluid of once-through steam generator based on three major conservation laws. The obtained models reflect dynamic characteristics of systems to a certain extent, but they are too complicated for controller design.

In view of applicable scope and limitations of existing research results, this paper combines the advantages of mechanism modeling and data modeling to model once-through steam generator. The main content is divided into two parts: (a) The first part is mechanism modeling: A linear model of once-through steam generator is established based on lumped parameter method; (b) Another is data modeling : The estimation method of unknown parameters is given, and actual operating data is used to verify the validity of proposed model and parameter estimation method. The paper is organized as follows: The model establishment is presented in Section 2, including introduction and mechanism modeling of once-through steam generator; The obtained model is described as a linear discrete form in Section 3. Meanwhile, its parameter estimation problem is transformed into total least squares problem and linear optimization problem with constraints, and their solutions are given respectively; In Section 4, the operating data of real system is used to verify the validity of proposed model and parameter estimation method given in this paper; Conclusion is given in Section 5.

parameters	descriptions
ρ_s	Steam density（kg/m^3）
ρ_w	Feed-water density（kg/m^3）
v	Flow rate（m/s）
$f(v)$	Friction between fluid（N）
P_w	Feed-water pressure（MPa）
P_s	Steam pressure（MPa）
V	Riser volume（m^3）
ε	Normalized length coordinates of riser
$x(\varepsilon)$	Vapor mass fraction at ε
α_m	Average steam ratio in riser
g	Gravitational acceleration（g/m^3）
m	Fluid mass in riser（kg）
A	Sectional area of OTSG（m^2）
Q_s	Mass flow rate of steam at outlet（kg/s）
Q	Heat input of OTSG secondary side（KJ）

2. MODELING

2.1. System Description

The object of this paper is a vertical once-through steam generator, and its structure comparison with natural circulation drum boiler is shown in **Fig.1**. For natural circulation steam drum boiler, feed water and drain flow into descending channel, and fluid in ascending channel gradually transfers into steam-water mixture under the heating of coolant in heat transfer tube. The force that drives fluid flow is formed by density difference between the two pipes. In contrast, the structure of once-through steam generator is relatively simple, without steam-water separator, which can basically be regarded as a riser. The feed water flows upward due to the pressure of feed water pump, and gradually forms steam-water mixture after being heated by heat in the primary circuit coolant. Finally all is converted into superheated steam at outlet. Obviously, the structure of once-through steam generator is similar to that of riser in drum boiler. In conclusion, the main differences between the two are following two points: (1) the ratio of steam at the inlet and outlet of the riser is different. There is a small amount of steam at the inlet of ascending riser in drum boiler, and proportion of steam at outlet is related to its cycle rate. For once-through steam generator, there is no steam at the inlet, and all is superheated steam at the outlet; (2) the driving force of fluid flow in drum boiler is determined by density difference between ascending channel and descending channel. However, fluid in once-through steam generator is driven by the pressure of feed water pump.

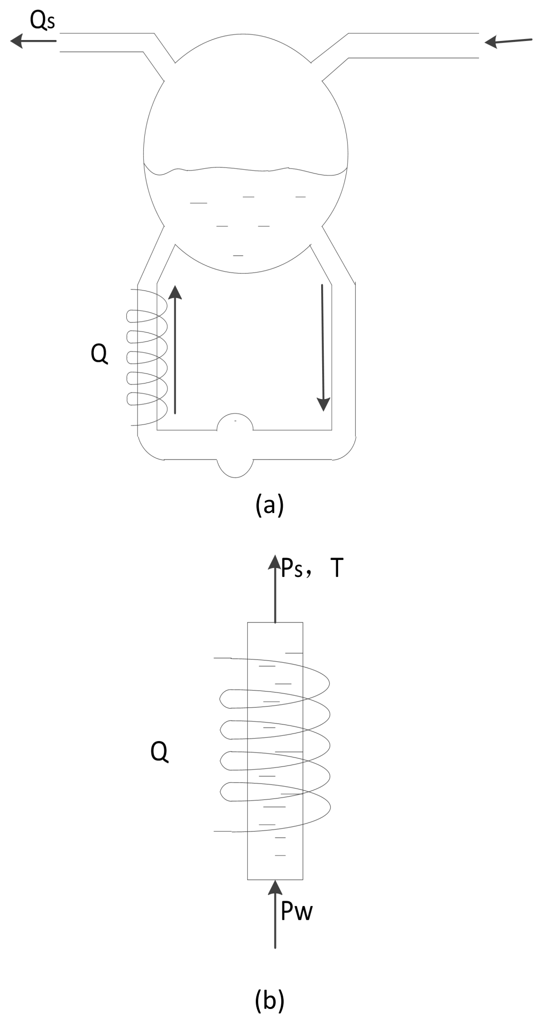


Fig. 1 Structure comparison between natural circulation drum boiler (a) and OTSG (b)

2.2. Mathematical Model

According to structure of once-through steam generator, lumped parameter method is used. Assuming that the flow rate of fluid in once-through steam generator is consistent, the momentum balance can be written as,

$$m\frac{dv}{dt} = A(P_w - P_s) - mg - f(v) \tag{1}$$

The friction between fluids can be expressed as,

$$f(v) = kv \tag{2}$$

In this equation, k is the correlation coefficient, v is the fluid velocity, and fluid friction is taken as a linear function of fluid velocity [5]. In addition, flow rate can be expressed by mass flow rate of outlet steam,

$$v = \frac{Q_s}{A\rho_s} \tag{3}$$

Substituting equation (2) and equation (3) into equation (1), there is,

$$m\frac{dQ_s}{dt} = A^2\rho_s(P_w - P_s) - A\rho_s mg - kQ_s \tag{4}$$

The superheated steam at outlet of once-through steam generator is transported to airtight pipeline. Then, according to ideal gas equation, there is,

$$P_sV_0 = nRT \tag{5}$$

In equation (5), V_0 is the volume of pipe, n is the number of moles of gas, R is the gas constant, and T is the gas temperature. Derivative of above equation with respect to time can be obtained,

$$\frac{dP_s}{dt} = \frac{RT}{V_0M}\cdot Q_s \tag{6}$$

M is the molar mass of gas molecule, and Q_s can be expressed as,

$$Q_s = \frac{V_0M}{RT}\cdot\frac{dP_s}{dt} \tag{7}$$

Substituting equation (7) into equation (4), the relationship between outlet steam pressure and feed-water pressure can be described as follows,

$$\frac{V_0M}{RT}\cdot m\cdot\frac{d^2P_s}{dt^2} + k\cdot\frac{V_0M}{RT}\cdot\frac{dP_s}{dt} + A^2\rho_sP_s = A^2\rho_sP_w - A\rho_s mg \tag{8}$$

Since fluid in once-through steam generator is two-phase, the proportion of steam and water will change with variables, then fluid mass--m is difficult to express. Therefore, based on analysis of natural

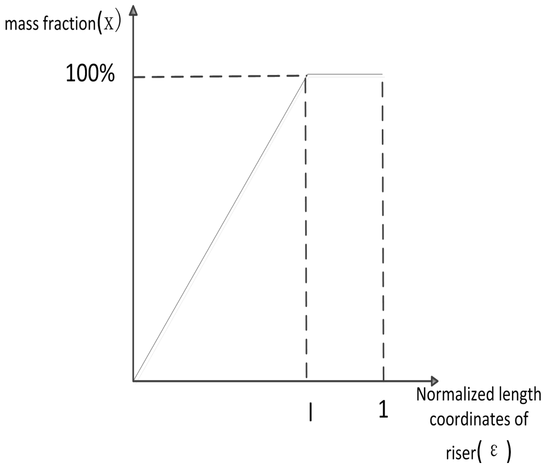


Fig. 2 Distribution of steam quality in OTSG

circulation drum boiler and once-through steam generator in previous section, an assumption in mathematical modeling of drum boiler is used for reference: steam is linearly and uniformly distributed in pipeline [6]. Based on this assumption, distribution of steam quality in once-through steam generator is shown in **Fig.2**. From l--the distance from the bottom of riser (normalized length), there is all superheated steam. And in 0~ l section, steam is distributed linearly. Steam mass fraction along the pipe in this segment is as follows [7],

$$x(\varepsilon) = \frac{\varepsilon}{l} \qquad 0 \le \varepsilon \le l \tag{9}$$

In 0~ l section, supposing when the proportion of steam is x, corresponding steam volume is α. There is,

$$x = \frac{\rho_s\alpha}{\rho_s\alpha + \rho_w(1-\alpha)} \tag{10}$$

Therefore,

$$\alpha = \alpha(x) = \frac{\rho_w x}{\rho_s + (\rho_w - \rho_s)x} \tag{11}$$

In l ~1 section, there is all steam. So the average volume ratio of steam in once-through steam generator is,

$$\alpha_m = \int_0^l \alpha(x(\varepsilon))\, d\varepsilon + 1 - l = 1 - l + \frac{\rho_w l}{\rho_w - \rho_s}\left[1 + \frac{\rho_s}{(\rho_w - \rho_s)}\ln\left(\frac{\rho_s}{\rho_w}\right)\right] \tag{12}$$

Thus, the fluid quality in once-through steam generator can be expressed as,

$$m = (\rho_w(1 - \alpha_m) + \rho_s\alpha_m)V \tag{13}$$

Substitute equation (13) into equation (8),

$$\frac{V_0M}{RT}\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)V\cdot\frac{d^2P_s}{dt^2}+k\cdot\frac{V_0M}{RT}\cdot\frac{dP_s}{dt}+A^2\rho_sP_s=A^2\rho_sP_w-A\rho_s\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)Vg \tag{14}$$

Equation (14) describes the relationship between feed water pressure and outlet steam pressure, supposing other variables such as heat input in the secondary side remain unchanged.

3. PARAMETER ESTIMATION

Unknown parameters of given model above can be estimated via traditional linear regression methods. Assuming that data sampling time is τ, the discretization mathematical model is,

$$A^2\rho_s\cdot P_{w_n}-A\rho_s\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)Vg=\frac{V_0M}{RT}\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)V\cdot\frac{P_{s_n}+P_{s_{n-2}}-2P_{s_{n-1}}}{\tau^2}+k\frac{V_0M}{RT}\cdot\frac{P_{s_n}-P_{s_{n-1}}}{\tau}+A^2\rho_s\cdot P_{s_n} \tag{15}$$

There are,

$$a_0=\frac{V_0M}{RT}\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)V \tag{16}$$

$$a_1=k\tau\cdot\frac{V_0M}{RT} \tag{17}$$

$$a_2=A^2\rho_s\tau^2 \tag{18}$$

$$a_3=A\rho_s\left(\rho_w(1-\alpha_m)+\rho_s\alpha_m\right)Vg\tau^2 \tag{19}$$

According to above equations, arrange equation (15) into the form of input- output,

$$P_{s_n}=\frac{2a_0+a_1}{a_0+a_1+a_2}P_{s_{n-1}}-\frac{a_0}{a_0+a_1+a_2}P_{s_{n-2}}+\frac{a_2}{a_0+a_1+a_2}P_{w_n}-\frac{a_3}{a_0+a_1+a_2} \tag{20}$$

In order to facilitate system identification, the equation (20) is organized into the following form,

$$P_{s_n}=\theta_0P_{s_{n-1}}+\theta_1P_{s_{n-2}}+\theta_2P_{w_n}+\theta_3 \tag{21}$$

Where,

$$\theta_0=\frac{2a_0+a_1}{a_0+a_1+a_2} \tag{22}$$

$$\theta_1=-\frac{a_0}{a_0+a_1+a_2} \tag{23}$$

$$\theta_2=\frac{a_2}{a_0+a_1+a_2} \tag{24}$$

$$\theta_3=-\frac{a_3}{a_0+a_1+a_2} \tag{25}$$

The measurable parameters are V_0, V, T and A, and the constant value is M and R.
Therefore, define the following linear model,

$$X_{(n-2)\times4}\triangleq\begin{bmatrix}x_3\\x_4\\\vdots\\x_n\end{bmatrix} \tag{26}$$

$$x_n\triangleq\left[\,P_{s_{n-1}}\quad P_{s_{n-2}}\quad P_{w_n}\quad 1\right] \tag{27}$$

$$Y_{(n-2)\times1}\triangleq\begin{bmatrix}y_3\\y_4\\\vdots\\y_n\end{bmatrix} \tag{28}$$

$$y_n\triangleq P_{s_n} \tag{29}$$

$$\theta_{4\times1}\triangleq\left[\theta_0\quad\theta_1\quad\theta_2\quad\theta_3\right]^T \tag{30}$$

The linear model is written as,

$$Y_{(n-2)\times1}=X_{(n-2)\times4}\theta_{4\times1} \tag{31}$$

Where, $\theta_{4\times1}$ is the column vector containing all parameters to be determined.
Considering measurement noise of input and output data, there is,

$$Y=\hat{Y}+E_Y \tag{32}$$

$$X=\hat{X}+E_X \tag{33}$$

Where, $\hat{Y}$ and $\hat{X}$ is actual measurement value, and E_Y and E_X is measurement noise. Then the outlet steam pressure model of once-through steam generator with measurement noise is expressed as,

$$\hat{Y}=(\hat{X}+E_X)\theta_{4\times1}+E_Y \tag{34}$$

Therefore, the parameter estimation problem is described as,

$$\arg\min \|[E_X, E_Y]\|_F \text{ s.t.}, \hat{Y} = (\hat{X} + E_X)\theta_{4\times1} + E_Y \quad (35)$$

Where, $\|\cdot\|_F$ is Frobenious norm.

The parameter estimation problem (35) is also called total least squares (TLS) problem. This method can ensure fitting error minimized. Under the condition that measurement noise is zero mean Gaussian noise, the algebraic solution of this problem is as follows,

$$\hat{\theta} = -V_{XY}V_{YY}^{-1}, \text{s.t.}, [\hat{X}, \hat{Y}]_{svd} = [U_X, U_Y]\begin{bmatrix} \sum X & \\ & \sum Y \end{bmatrix}\begin{bmatrix} V_{XX} & V_{XY} \\ V_{YX} & V_{YY} \end{bmatrix} \quad (36)$$

In equation (36), $[\hat{X}, \hat{Y}]_{svd}$ means to perform singular value decomposition on extended matrix--$[\hat{X}, \hat{Y}]$.

Taking into account that some assumptions are used in mechanism modeling, with there being some measurement errors in parameter estimation, the estimation results obtained by total least square method may have large deviations or be inconsistent with physical meaning. Therefore, linear optimization method is also used to estimate parameters. The problem then can be described as follows,

$$\exists x \in R^n \text{ s.t. } \min(f(x)) \quad (37)$$

Where, $f(x)$ is a linear function of variable x. And there are some constraints, so the range of variable is limited by linear conditions.

$$\min \quad f(\theta) = \|Y - X\theta\|_2 \quad (38)$$

$$\text{s.t. } \theta_0 + \theta_1 + \theta_2 = 1, \quad (39)$$

$$I * [\theta_0 \quad -\theta_1 \quad \theta_2 \quad -\theta_3]^T \geq 0 \quad (40)$$

$\|\cdot\|_2$ is 2 norm. According to above equations, parameters can also be estimated by linear optimization method.

4. CALCULATION RESULTS

In this section, real operating data of once-through steam generator is used to verify the validity of proposed model (14) and parameter estimation(35,37).

Firstly, use 1300 sets of real operating data (sampling time = 1s) to estimate parameters of proposed model, and take the average of multiple estimations as final result; Then use another 1300 sets of data to verify the estimated results. In view of the fact that once-through steam generator system studied by this paper has no real reference, predicted steam pressure by proposed model is used to be compared with actual measured steam pressure during verification process.

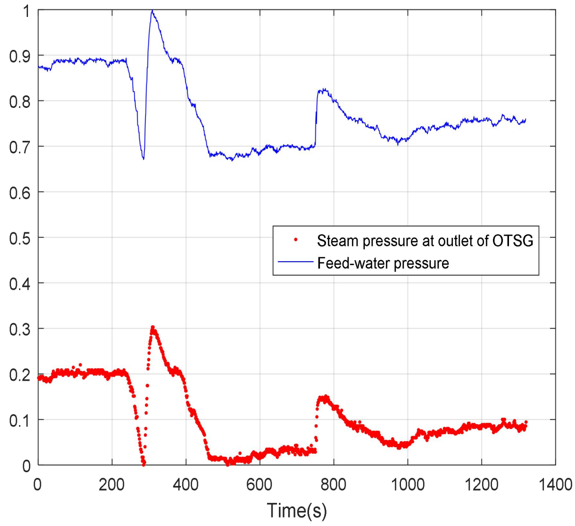


Fig. 3 The actual operating data of OTSG

The actual operating data used for parameter estimation is shown in **Fig.3**. Here, data is normalized to shield actual operating state of system. This operation does not affect parameter estimation and result verification.

The first 1300 sets of real data are divided into 10 groups, and take each 9 groups every time for parameter estimation, which is performed 10 times in total. Then, take the average value of parameter estimations for 10 times as the final result. Parameter estimation results obtained by total least square method are shown in **Table.1**. Following the same steps, estimation results via linear optimization are shown in **Table.2**.

Table. 1 Parameter estimation results by total least square

Unknown parameters	Mean value $\mu(\hat{\theta})$	Standard deviation $std(\hat{\theta})$	95% confidence interval for mean value	95% confidence interval for Standard deviation
$\hat{\theta}_0$	-3.6499×10^{-2}	4.5017×10^{-2}	$[-7.3201, 0.0204] \times 10^{-2}$	$[3.2252, 9.1474] \times 10^{-2}$
$\hat{\theta}_1$	2.8563×10^{-1}	1.4548×10^{-2}	$[2.7377, 2.9749] \times 10^{-1}$	$[1.0423, 2.9561] \times 10^{-2}$
$\hat{\theta}_2$	6.8573×10^{-1}	3.4894×10^{-2}	$[6.5728, 7.1418] \times 10^{-1}$	$[2.4999, 7.0904] \times 10^{-2}$
$\hat{\theta}_3$	1.9208×10^{-2}	3.5015×10^{-3}	$[1.6353, 2.2063] \times 10^{-2}$	$[2.5086, 7.1149] \times 10^{-3}$

Table. 2 Parameter estimation results by linear optimization

Unknown parameters	Mean value $\mu(\hat{\theta})$	Standard deviation $std(\hat{\theta})$	95% confidence interval for mean value	95% confidence interval for Standard deviation
$\hat{\theta}_0$	4.8030×10^{-1}	1.3141×10^{-1}	$[3.8121, 5.7939] \times 10^{-1}$	$[0.9528，2.5288] \times 10^{-1}$
$\hat{\theta}_1$	-1.8173×10^{-3}	9.5546×10^{-4}	$-[2.5378, 1.0969] \times 10^{-3}$	$[0.6927, 1.8386] \times 10^{-3}$
$\hat{\theta}_2$	5.2152×10^{-1}	1.3108×10^{-1}	$[4.2268, 6.2036] \times 10^{-1}$	$[0.9504, 2.5225] \times 10^{-1}$
$\hat{\theta}_3$	-1.2308×10^{-1}	3.0872×10^{-2}	$-[1.4636, 0.9980] \times 10^{-1}$	$[2.2384, 5.9409] \times 10^{-2}$

According to comparison of results in **Table.1 and 2**, the variance of estimation results via TLS method is small and fitting effect is better, but the parameters do not conform to physical meaning. In **Table.1**, The sign of results do not accord with actual parameters. Fitting effect of linear optimization method is slightly worse, but it fully meets the demands of controller design, and the estimated results conform to physical meanings well. Therefore, a good identification model can be obtained with results in Table 2. The effectiveness can be evaluated through its fitting effect to real data, which is shown in **Fig.4**.

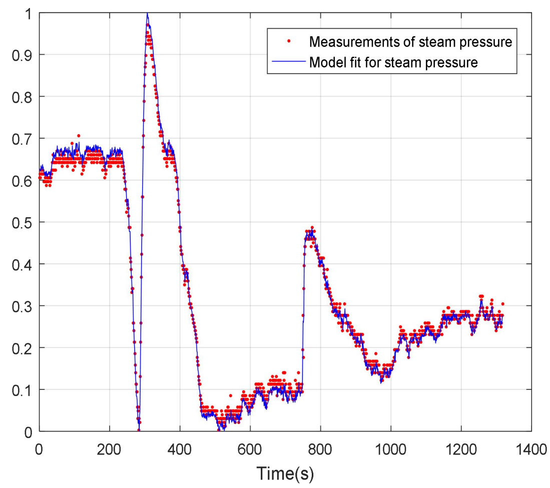


Fig. 4 The fitting effect of identified model to real data

Fig.4 reflects fitting effect of normalized data, so maximum fitting error is relatively large. Define average relative error and maximum relative error of model fitting effect to real operating data as follows:

$$\mu(\widehat{\varepsilon_n}) \triangleq \frac{\mathrm{mean}(\|\hat{P}_{s_n}-P_{s_n}\|)}{\max(P_{s_n})} \quad (41)$$

$$\max(\widehat{\varepsilon_n}) \triangleq \frac{\max(\|\hat{P}_{s_n}-P_{s_n}\|)}{\max(P_{s_n})} \quad (42)$$

The fitting error of proposed model is,

$$\mu\left(\widehat{\varepsilon_n}\right) = 0.05\%, \max\left(\widehat{\varepsilon_n}\right) = 0.32\% \quad (43)$$

Another 1,300 sets of data are used to verify the validity of proposed model. The difference between predicted value by identified model and output value of real system reflects the effect of proposed model on real system. **Fig. 5**. shows the comparison of data.The model given in this paper is a dynamic model in the form of "input-output", so the output depends on historical state. According to source of historical state, there are two cases. 1-step prediction: historical state is measurement value of real system; n-step prediction: historical state is predicted value of proposed model (except the initial state). With actual measurement data used for prediction each step, The 1-step prediction essentially reflects the static modeling effect; In n-step prediction, proposed model operates with the same initial state as real system independently, which reflects the dynamic modeling effect [8].

Fig.5(a) shows result of 1-step prediction. The actual measured value-- $P_{s_{n-1}}$, $P_{s_{n-2}}$ is used to predict current output. The average relative prediction error and maximum relative prediction error are as follows,

$$\mu\left(\widehat{\varepsilon_{1,n}}\right) = 0.04\%, \max\left(\widehat{\varepsilon_{1,n}}\right) = 0.24 \quad (44)$$

Fig.5(b) shows results of 1300 step prediction. With initial actual value-- P_{s_1} , P_{s_2} , output-- $\hat{P}_{s_n}$ is independently calculated based on input data obtained by identification model, n = 3,..., 1300. Its average relative prediction error and maximum relative prediction error are as follows,

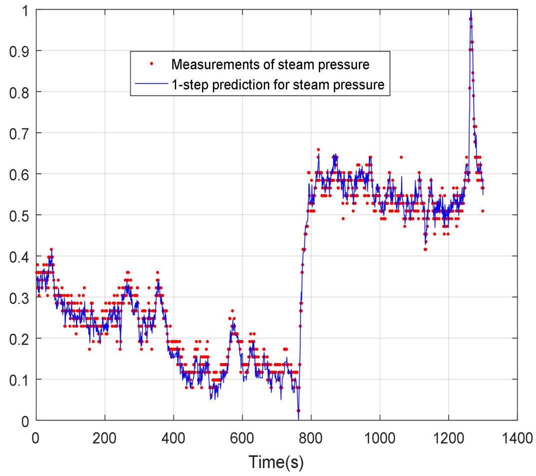
$$\mu\left(\widehat{\varepsilon_{100,n}}\right) = 0.05\%, \max\left(\widehat{\varepsilon_{100,n}}\right) = 0.29 \quad (45)$$

The effect of 1-step prediction is slightly better than 1300-step prediction, which essentially stems from cumulative measurement errors. The variance of prediction increases as the number of model iteration step gets more [8]. However, the difference between results of the two predictions is small, which proves that the parameters and model obtained in this paper are relatively accurate. The error of each step is basically negligible, so cumulative error is also small.In conclusion, overall forecast is very good.

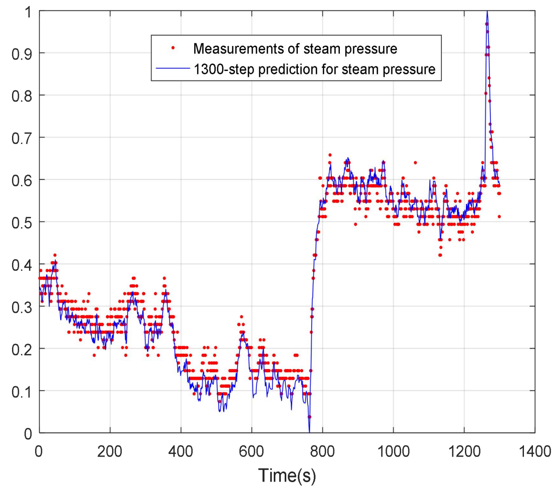
5. CONCLUSION

In this paper, pressure model of once-through steam generator is identified. Model structure and unknown parameters of the system are determined by mechanism modeling and data modeling. Dynamics of

Fig. 5 The comparison of measured data and prediction value for



(a)



(b)

outlet steam

once-through steam generator are described in the form of "input-output", which is suitable for application and research such as controller design and fault diagnosis. Parameters of identification model are estimated by linear optimization and total least squares method, which can be applied for the case where input and output measurement contains noise. What should be noted is that the model given in this paper is mainly used to study relationship between feed-water pressure and outlet steam pressure of once-through steam generator. In this condition, other relative variables are regarded as constants. Therefore, appropriate adjustments should be made to the model based on other mechanism information during actual applications.

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