

Paper:

Perimeter Control for Macroscopic Fundamental Diagram Systems Based on Neural Network and Generalized Predictive Control

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Abstract. Macroscopic traffic flow forms show heavily similarity between days, so this paper uses past datas to train a neural network for fitting the MFD system. This paper presents a framework for combining neural network with generalized predictive control. First, a region traffic network model is identified by the neural network which can approximate any nonlinear system. Second, the MFD system is transformed into linear system because of linearizing neural network models at each operation point. Third, the optimal perimeter control of the two regions is achieved by the generalized predictive control method with the instantaneous linearization model. Simulation results show that the proposed frame significantly alleviates congestion in the network and reduces the total travel time spend in the traffic network.

Keywords: Macroscopic fundamental diagrams, Neural network, Generalized predictive control, Perimeter control.

1. Introduction

Urban traffic congestion is a challenging problem. Automatic control method has been an effect way to solve the problem. Some signal control strategies have widely used such as SCOOT (split cycle offset optimizing technique) [1] and SCATS (sydney coordinated adaptive traffic system) [2] in single point control and trunk line control and they have achieved good results. However, those control strategies are less efficient under oversaturated traffic conditions. Traffic control issues need to be reconsidered from a network-level traffic models because of traffic congestion caused by a surge of vehicles. So Godfrey (1969) [3] presented macroscopic fundamental diagrams (MFD). The existence of MFD was verified by data of Yokohama in 2008 [4]. The MFD is a function describing the nonlinear relationship between the regions accumulation $n_i(t)$ and the trip completion rate $G_i(n_i(t))$ (veh/s) (Fig. 1). Traffic network control at the macro level is feasible according to the proof.

The perimeter control with MFD is introduced to reducing congestion. Daganzo firstly introduced perimeter

flow control policies to single region [5]. Then, Haddad and Geroliminis extended optimal perimeter control to two regions [6]. A nonlinear model used to designing optimal multivariable integral feedback regulators was introduced to describes the evolution of the multi-region system [7]. Perimeter control based on linear quadratic regulator (LQR) was used to two region at an equilibrium point by Geroliminis (2013) [8]. Mehmet Yildirimoglu proposed equilibrium analysis and route guidance in large-scale networks with MFD dynamics [9]. Then, researchers considered time-delay and robust in order to improve control effect. Robust control framework was concerned the integration of a bi-modal Macroscopic Fundamental Diagrams (MFD) modelling for mixed traffic for congested single-region and multi-region urban network [10]. Mehdi Keyvan-Ekbatani designed controller for gating traffic control with time-delay in urban road networks [11]. Haddad considered robust constrained control of uncertain macroscopic fundamental diagram networks [12].

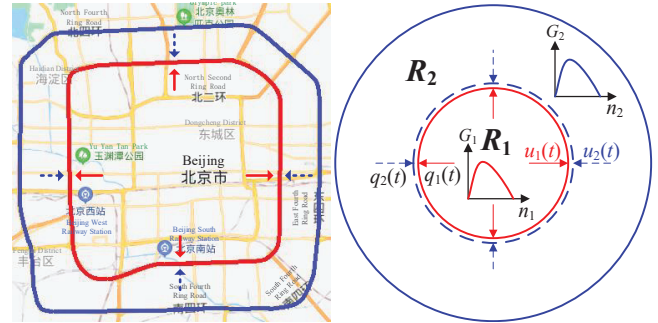


Fig. 1. Two-region MFD system.

Scholars tried to apply the model predictive control (MPC) for optimal perimeter control, because MPC is a predictive control algorithm with feedback correction and open-loop rolling optimal. Geroliminis proposed a model predictive approach for two urban regions macroscopic fundamental diagrams [13]. Jack Haddad used MPC to cooperative traffic control of a mixed network with two urban regions and a freeway [14]. Earlier works on perimeter control-based MPC schemes, MFD-based economic MPC schemes to improve mobility used in heterogeneously congested large-scale urban road net-

works [15]. Stochastic Model Predictive Control (SMPC) [16] and nonlinear model predictive perimeter control approaches [17] are employed for optimal perimeter control of traffic flow with MFD. Above papers from different angles to study shows that researchers are aware of that there are many factors lead to some error between region perimeter control design and MFD dynamic model. however, the problem is not completely solved. So a neural network approach approximate to MFD dynamic system is proposed in this paper to reduce the error.

The rest of the paper is organized as follows: In Section 2, the two-region MFD dynamics is introduced, and neural network is conducted to approximate two-region MFD dynamic system. In Section 3, Instantaneous linearization model is extracted from neural network model which is trained in section 2. Generalized predictive control method is used to give the optimal law in Section 4. Section 5 presents the numerical experiments. Section 6 concludes the paper.

2. Two-Region MFD Identification Based on Neural Network System

In this section, traffic network dynamic model for the two regional MFD system with input and state constraints is recapitulated. A neural network is constructed to approximate the dynamic nonlinear MFD systems.

2.1. Two-Region MFD Dynamic Systems

Two-Region perimeter control has certain application environment, such as the inner area and outer area of the third ring in Beijing (Fig. 1). Two-region perimeter control is to control the transfer flow between two regions with MFD system and maximize the total number of vehicles finished traveling in the two regions. The perimeter controller only controls the transmission flow between regions rather than controls the traffic flow within regions.

Two-region dynamic evolution of traffic system is [18]:

$$\begin{aligned} \dot{n}_1(t) = & -M_{11}(t)G_1(n_1(t)) - \\ & M_{12}(t)G_1(n_1(t))u_1(t) + \dots \quad (1) \\ & M_{21}(t)G_2(n_2(t))u_2(t) + q_1(t) \end{aligned}$$

$$\begin{aligned} \dot{n}_2(t) = & -M_{22}(t)G_2(n_2(t)) - \\ & M_{21}(t)G_2(n_2(t))u_2(t) + \dots \quad (2) \\ & M_{12}(t)G_1(n_1(t))u_1(t) + q_2(t) \end{aligned}$$

subject to

$$M_{11}(t) + M_{12}(t) = 1, \quad M_{21}(t) + M_{22}(t) = 1 \quad (3)$$

$$M_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)}, \quad M_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)} \quad (4)$$

$$0 < n_1(t) < n_1^{\text{jam}}, \quad 0 < n_2(t) < n_2^{\text{jam}} \quad (5)$$

$$0 \leq u_1(t) \leq 1, \quad 0 \leq u_2(t) \leq 1 \quad (6)$$

where $n_i(t)$, $i = 1, 2$ (veh) is accumulation of region i at time t , $u_i(t)$ is perimeter control input given by perimeter

control denote the percentage of flow allowed to transfer from region i to region j . $G_i(n_i(t))$ is completed transfer flow, $(n_{ij}(t)/n_i(t))G_i(n_i(t))$ is the number of vehicles from region i to region j , and the transfer flow completed after being controlled by the perimeter controller $u_i(t)(n_{ij}(t)/n_i(t))G_i(n_i(t))$. n_1^{jam} and n_2^{jam} are the jammed accumulations in R_1 and R_2 , respectively. M_{ii} denotes the ratio of traffic flow from region i to region i . M_{ij} denotes the ratio of traffic flow from region i to region j . $q_1(t)$ is the traffic demand and the traffic flow out of control for R_1 region, $q_2(t)$ is the traffic demand and the traffic flow out of control for R_2 region.

2.2. Two-Region MFD Identification Based on Neural Network System

The neural network which belongs to the global approximation method has ability to approximate any nonlinear mapping relation and has good generalization ability. Thus we use three-layer neural network to identify the two-region MFD dynamic system as shown in Fig. 2.

$n_1(t-1)$, $n_1(t-2)$, $n_2(t-1)$, $n_2(t-2)$, $u_1(t-1)$, $u_1(t-2)$, $u_2(t-1)$, $u_2(t-2)$ denote the input layer of the neural network, while $n_1(t)$, $n_2(t)$ denote the outputs layer of the neural network. (w , W) are the adjustable parameter weights of the network, and they are trained from the traffic data ($n_i(t)$, $u_i(t)$, $i = 1, 2$). The “state” $x(t)$ is then introduced as a vector composed of inputs. Specify the training set by:

$$x(t) = \{[u_i(t), n_i(t)] \mid i = 1, 2\} \quad (7)$$

The network will predict accumulation vehicles $\hat{n}_i(t)$, which is close to the true accumulation $n_i(t)$. Neural network of two-region MFD system has one hidden layer and the weight between layers are (w , W) and the hyperbolic tangent and liner activation functions (f , F) [28]:

$$\begin{aligned} \hat{n}_i(t) = & F_i \left(\sum_{j=1}^k W_{ij} h_j(w) + W_{i0} \right) \\ = & F_i \left(\sum_{j=1}^k W_{ij} f_j \left(\sum_{l=1}^p w_{jl} x_l + w_{j0} \right) + W_{i0} \right), \end{aligned} \quad (8)$$

The approximate model here may be interpreted as a linear model extracted from the neural network with one hidden layer of tanh units and a linear output. The approximate model is

$$\hat{n}_i(t) = \sum_{j=1}^k W_j \tanh \left[\sum_{l=1}^p w_{jl} x_l + w_{j0} \right] + W_{i0}. \quad (9)$$

The mapping (9) to be:

$$\hat{n}_1(t) = g_1(N(t-1), N(t-2), U(t-1), U(t-2)) \quad (10)$$

$$\hat{n}_2(t) = g_2(N(t-1), N(t-2), U(t-1), U(t-2)) \quad (11)$$

where $N(t) = [n_1(t), n_2(t)]^T$, $U(t) = [u_1(t), u_2(t)]^T$. The mapping is trained by neural network from $x(t)$ to $\hat{n}_i(t)$, $i = 1, 2$.

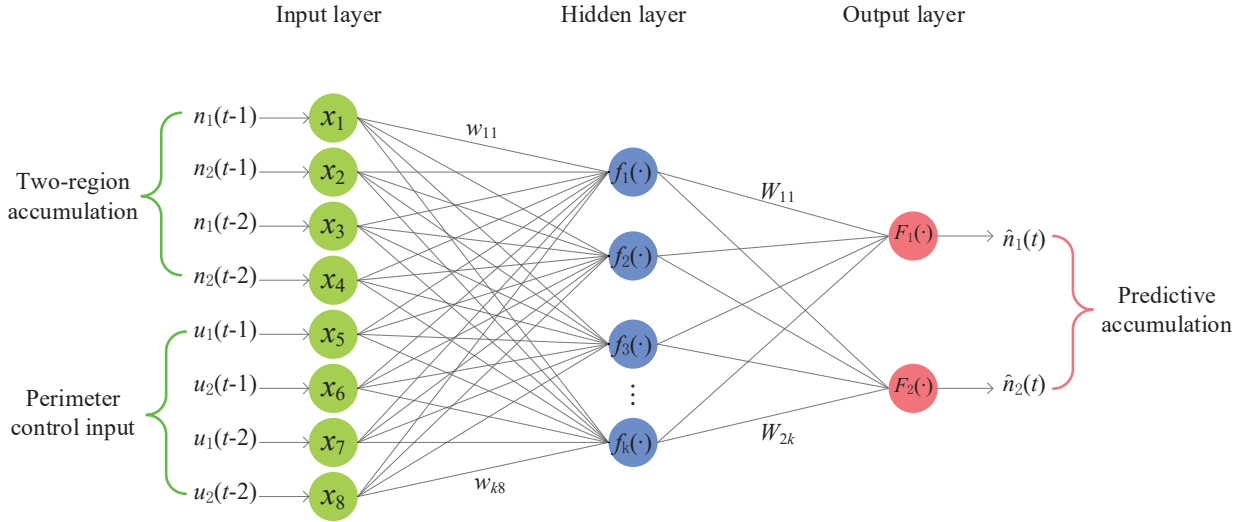


Fig. 2. Three-layer neural network for two-region MFD approximation.

3. Instantaneous Linearization Model of Two-Region MFD system

In this section, two-region MFD system controller based on generalized predictive control and instantaneous linearization model extracted from neural network is proposed. First, neural network is used to approximate non-linear two-region MFD system. Second, a linear model is extracted from the neural network model. Third, a predictive controller is designed at each sample to controls the two-region MFD system [19]. See the Fig. 3 for details.

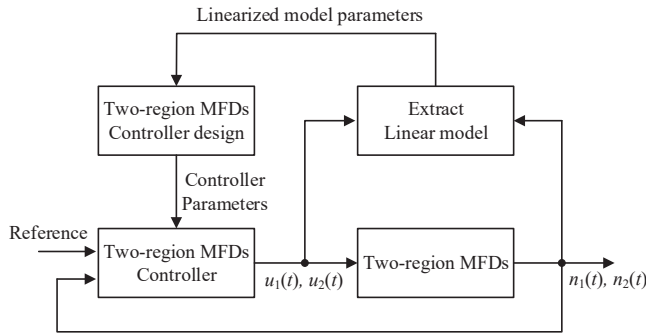


Fig. 3. Two-Region MFD control system design.

The generalized predictive control method was proposed by Clarke, and Controlled Auto-Regressive Integrated Moving Average (CARIMA) model was used as prediction model [20]. It has three basic characteristics: prediction model, rolling optimization and feedback correction, and presents excellent control performance and robustness. Extracting linearized model Eq. (9-11) becomes to be :

$$\begin{aligned} \tilde{N}(t) = & -A_1 \tilde{N}(t-1) - A_2 \tilde{N}(t-2) + \\ & B_0 \tilde{U}(t-1) + B_1 \tilde{U}(t-2) \end{aligned} \quad \dots \quad (12)$$

$$\begin{aligned} A_i = & \begin{bmatrix} -\frac{\partial g_1(x(t))}{\partial n_1(t-i)} \Big|_{x(t)=x(\tau)} & -\frac{\partial g_1(x(t))}{\partial n_2(t-i)} \Big|_{x(t)=x(\tau)} \\ -\frac{\partial g_2(x(t))}{\partial n_1(t-i)} \Big|_{x(t)=x(\tau)} & -\frac{\partial g_2(x(t))}{\partial n_2(t-i)} \Big|_{x(t)=x(\tau)} \end{bmatrix} \\ i = 1, 2, \\ B_i = & \begin{bmatrix} -\frac{\partial g_1(x(t))}{\partial u_1(t-i)} \Big|_{x(t)=x(\tau)} & -\frac{\partial g_1(x(t))}{\partial u_2(t-i)} \Big|_{x(t)=x(\tau)} \\ -\frac{\partial g_2(x(t))}{\partial u_1(t-i)} \Big|_{x(t)=x(\tau)} & -\frac{\partial g_2(x(t))}{\partial u_2(t-i)} \Big|_{x(t)=x(\tau)} \end{bmatrix} \\ i = 0, 1, \end{aligned}$$

where $N(t) = [n_1(t), n_2(t)]^T$, $U(t) = [u_1(t), u_2(t)]^T$.

4. Perimeter Controller Design of Two-Region MFD

The approximate model may thus be interpreted as a linear model. The criterion is to maximize the output of the traffic network. Therefore, the two-region MFD control problem is described as follows:

$$\begin{aligned} J(t, U(t)) = & \sum_{i=N_1}^{N_2} [r(t+i) - \hat{N}(t+i)]^2 + \\ & \lambda \sum_{i=1}^{N_u} [\Delta U(t+i-1)]^2 \end{aligned} \quad \dots \quad (13)$$

subject to

$$U(t) = [U(t) \dots U(t+N_u-1)]^T \quad \dots \quad (14)$$

$$\Delta u(t+i) = 0, \quad N_u \leq i \leq N_2 - 1 \quad \dots \quad (15)$$

$$0 < n_1(t) < n_1^{\text{jam}}, \quad 0 < n_2(t) < n_2^{\text{jam}} \quad \dots \quad (16)$$

$$0 \leq u_1(t) \leq 1, \quad 0 \leq u_2(t) \leq 1, \quad \dots \quad (17)$$

Where N_1 is the minimum prediction horizon, N_2 is the maximum prediction horizon, and N_u is the control horizon. $\hat{N}(t+i)$ are determined as the minimum variance predictions. λ denotes a weight factor for penalizing vari-

ations in the control input. The optimization problem results in a sequence of future controls $U(t)$. According to the principle of rolling optimization, the optimal control $U(t)$ denotes calculated by minimizing the objective function J .

5. Case Studies

The major objective of perimeter control is to operate the network traffic state in the uncongested regime. Four experiments are carried out to illustrate the effectiveness of the proposed method in this paper.

This paper considers a heterogeneous traffic network which can be divided into two homogeneous regions. A traffic network with a two-region system is illustrated.

In example 1, both regions R_1 and R_1 are initially in the uncongested regime, initial accumulation $[n_1(0), n_2(0)] = [3000, 1800]^T$ (veh). In addition, the traffic demand is set to be constant. Fig. 4 shows that the accumulation of region R_1 is decreasing while the accumulation of region R_2 is increasing, in the beginning of the control process. Later, the cumulants of the two regions approach each other. Fig. 5 presents the control input u_1 and u_2 . This result shown the effective control when the both regions are uncongested.

In example 2, regions R_1 is initially congested and R_2 is initially uncongested, initial accumulation is $[n_1(0), n_2(0)] = [4000, 1800]^T$ (veh). In addition, the traffic demand is set to be constant. Fig. 6 shows that the accumulation of region R_1 drops significantly at the first 5 min and the accumulation of two region are uncongested for the rest of the time. This example shows that the controller is sensitive to the congested traffic condition, and it effectively eases traffic congestion. And the control inputs are shown in Fig. 7. This result shown the control effective when one region is uncongested and another region is congested.

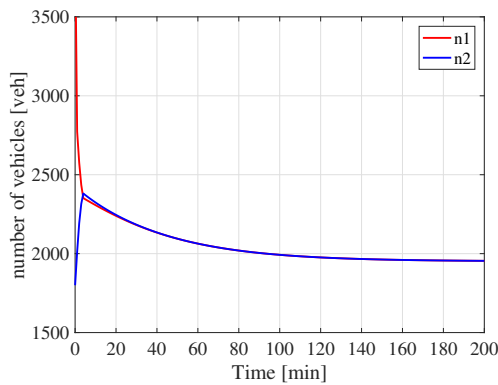


Fig. 4. Example 1. The number of vehicles in two-region.

In example 3, the accumulations of both regions are in the congested. Initial accumulation is $[n_1(0), n_2(0)] = [5000, 4000]^T$ (veh). Due to the congestion, the trip completion rate is low at the beginning, so it takes a longer

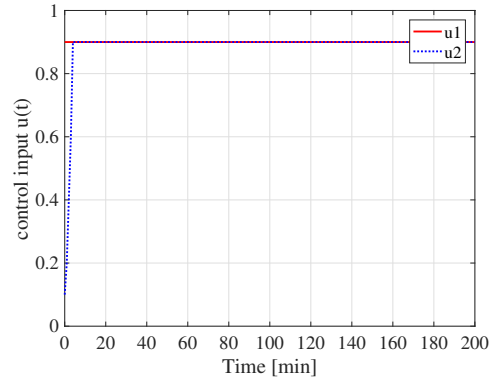


Fig. 5. Example 1. The control inputs of two-region.

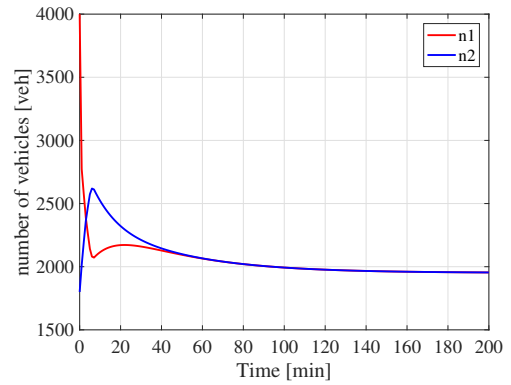


Fig. 6. Example 2. The number of vehicles in two-region.

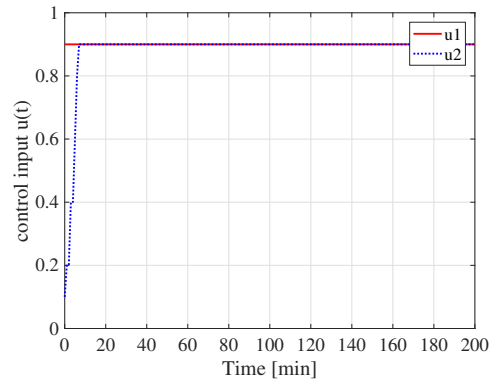


Fig. 7. Example 2. The control inputs of two-region.

time to be uncongested, compared to the two experiments above, the process is shown in Fig. 8. Then, as congestion is eliminated, the accumulations decline is faster, thanks to an increase in the completion rate. And the control inputs are shown in Fig. 9. The Controller alleviates congestion in this examples.

In example 4, regions R_1 is initially congested and R_2 is initially uncongested, initial accumulation is $[n_1(0), n_2(0)] = [5000, 3000]^T$ (veh). The demand shown

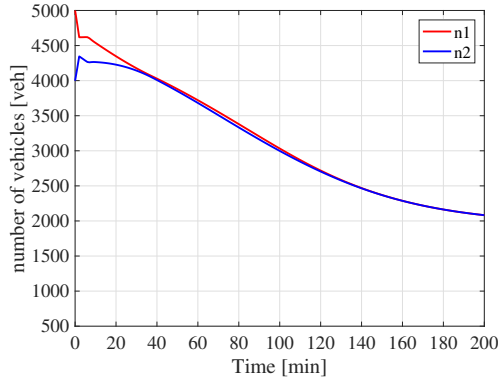


Fig. 8. Example 3. The number of vehicles in two-region.

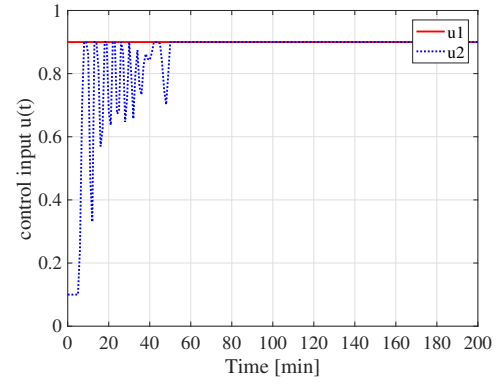


Fig. 11. Example 4. The control inputs of two-region.

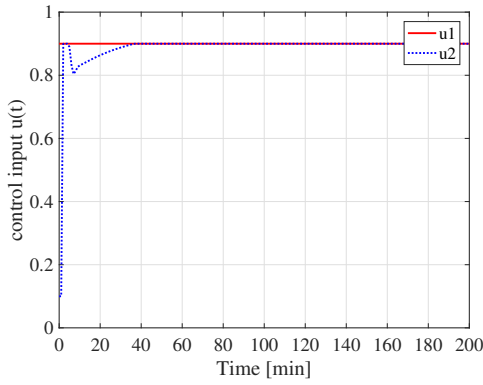


Fig. 9. Example 3. The control inputs of two-region.

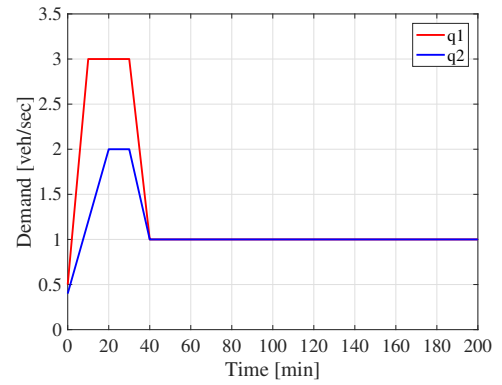


Fig. 12. Example 4. The demand of two-region.

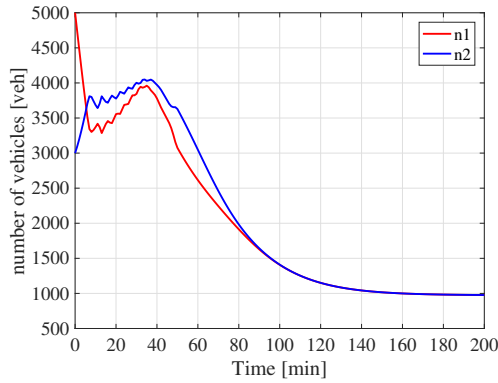


Fig. 10. Example 4. The number of vehicles in two-region.

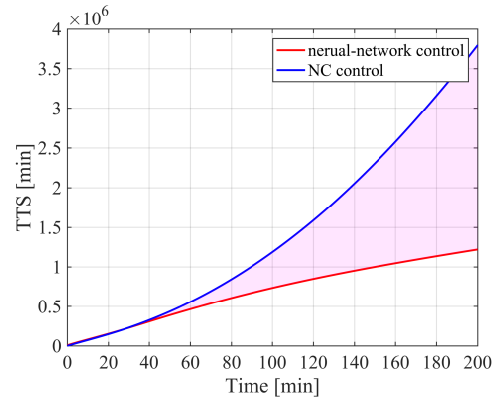


Fig. 13. Example 4. Total travel time spend of neural network and no control.

in Fig. 12 is high and time-varying. This example simulates the situation of high demand in congestion, such as morning peak and evening peak in a day.

From Fig. 10, the accumulation of the two regions increased in the first 40 minutes due to the high demand in the first half hour. Later, as demand decreased, so did the accumulations in both regions. Of course, the role of the controller is indispensable here, and the specific control input is shown in Fig. 11. From Fig. 13, we notice

the advantage of the neural network controller compared to the no control according to the total travel time spent (TTS)(min). Moreover, the pink region in Fig. 13 is the time saved by neural network controller, it shows that the control method proposed in this paper has a very good effect.

6. Conclusions

In this paper, the perimeter control problem for a two-region network was solved by neural network and generalized predictive control. First, two-region network systems was identified by neural network. Second, using linearizing neural network models at each operation point, the MFD system had been transformed into linear system. Third, two-region network systems was controlled by generalized predictive control method with the instantaneous linearization model. From the simulation examples conducted on a two-region MFD network, neural network and generalized predictive controller had an obvious effect on regional perimeter control, and total travel time spent have an obvious decrease.

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