Paper:

Multi-objective Path Planning Based on An Improved GWO-WOA Method

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The problem of multi-objective path planning for robots is to find the shortest, smoothest and safest path in the presence of obstacles, which can be proposed as a multi-objective optimization problem with generous constraints. In this paper, an improved GWO-WOA method is proposed achieve the solution, which mainly consists the following two improvements: First, a tent chaos mechanism is used to improve the initial population quality of grey wolf optimizer (GWO). Second, the hunting mechanism of the whale optimizer algorithm (WOA) is replaced to the hunting strategy of GWO, which can improve the tracking performance of the optimal global exploration searching solutions and avoid falling into the local optimization. Then, a smooth path is calculated based on the proposed hybrid GWO-WOA with spline interpolation method. To further reduce the computing burden, an one-dimensional search method for iteration process is proposed and compared with two-dimensional search method. Finally, simulation experiments demonstrate the feasibility and effectiveness of the proposed algorithm under different environments.

Keywords: Path planning, Tent chaotic strategy, GWO-WOA method, Collision avoidance, Path smoothness

1. Introduction

Nowadays, mobile robots are widely used in the field of manufacturing industries, mining, rescuing, military, agriculture, aerospace, just to name a few. Path planning as one of the key tasks for mobile robots motion planning, has been widely investigated by scholars[1, 2]. Usually, there are various paths for a robot to reach one goal point from one start point, and the goal of path planning is to find a feasible path satisfying certain constraints in a partially known environment. Based on the existing results, path planning can be divided into two categories: local path planning and global path planning[3]. In local path planning, robots have limited knowledge about environments, while in global path planning, they have a whole environment knowledge and can reach the target from a predefined path[4].

Since the late 1960s, a great number of path planning strategies have been published, including classical approaches (such as cell decomposition, roadmap approach, artificial potential field based algorithm), probabilistic based method, fuzzy logic and neural network algorithm, etc. However, the above mentioned methods may be inefficiency due to high computational cost and inaccuracy. Since path planning can be treated as an optimization problem, recently decades, with the development of swarm intelligence algorithm[5], many swarm intelligence methods, such as ant colony optimization (ACO), particle swarm optimization (PSO), artificial bee colony (ABC), cuckoo search (CS), grey wolf optimizer (GWO), whale optimizer algorithm (WOA) and so on, have been considered as a reliable solution of the path planning problem, due to that these swarm intelligent methods can yield effective, accurate and rapid solutions[6–9].

Since each swarm intelligence method may have its own drawbacks, but they can sometimes complement these drawbacks by each other. Recent years, hybrid algorithms by combining multiple swarm intelligence methods for better optimization solutions have been studied extensively. In [10], a novel hybrid PSO-MFB algorithm by combining PSO and modified frequency bat (MFB) is proposed and used for path planning of an autonomous mobile robot in dynamic environments. In [11], a hybridized path planning algorithm is designed based on bare-bones particle (BBO) and differential evolution.

Among the existing swarm intelligence techniques, GWO is originally presented by Mirjalili in 2014[12], which is motivated by the hunting process of grey wolves in nature. Superior to other meta-heuristic algorithms, GWO has the advantages of simplicity, flexibility and local optima avoidance, it has been widely applied to the issues of workshop scheduling, image classification, and parameter optimization. Based on GWO, path planning problem can be encoded as the best position of wolves. But it also faces the problems of easily falling into local optimum, low accuracy and slow convergence speed. Since swarm intelligence algorithms need to keep an excellent balance between the exploration and exploitation for achieving the global and local searches efficiently, by analysis, it is noticed that the optimization pattern of GWO has great exploitation capacity, while the WOA owns an aptitude for local exploitation ability. Meanwhile both the GWO and WOA show the possibility of boosting per-

M. Zhou is thankful for the grant funded by NSFC (51805021) and Research foundation for talents of NCUT (No.213051360020XN173/017, 107051360021XN090/010); J. Wang is thankful for the grant funded by NSFC (61973023) and Beijing Natural Science Foundation (4202052)

formance compared with other swarm algorithms, which motives the work of this paper by combining the GWO and WOA methods.

The main contributions of this paper consist the following aspects:

1. An improved hybrid GWO-WOA method is introduced to proposed, in which a tent chaos mechanism is improve the initial population quality of grey wolf optimizer and the hunting mechanism of the WOA is replaced to the hunting strategy of GWO.

2. The proposed hybrid GWO-WOA method is applied into the path planning problem which is used to find some important mid-points of the path, then based on these points a smooth route is generated with spline interpolation technique.

3. To reduce the computing burden, one-dimensional search method is proposed with the aid of coordinate system transformation technique, simulation results are demonstrated the effectiveness of the proposed method by comparing with the two-dimensional search method.

The following structure of this paper is organized as follows. In Section 2, the problem statements and preliminaries are discussed. In Section 3, an improved hybrid GWO-WOA is designed. In Section 4, the smooth path planning problem based on the proposed GWO-WOA is investigated. Then experimental results are conducted in Section 5. Finally, In section 6, some conclusions of this work is given.

2. Problem Statement and Preliminaries

Path planning problem aims to fine a collision-free optimized route from one given start point to the goal point with minimal comprehensive costs, i.e. safety, shortest distance, path smoothness, shortest time, minimal energy consumption, etc. In this paper, we mainly focus on two performance indices: shortest distance and path smoothness.

Considering that there are several mid-points between the star point (x_0, y_0) and the end point (x_D, y_D), such as the total path is described as $(x_0, y_0), (x_1, y_1), \cdots, (x_i, y_i), \cdots, (x_D, y_D)$, then the total path length dis is the sum of all distances between all the points including the state point, mid-points and end point[10], which is described as:

$$dis = \sum_{i=0}^{D} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad . \quad . \quad . \quad . \quad (1)$$

After these points are obtained, the path can be generated by connecting these points. However the path will be not smooth if there are only several mid-points, which is not applicable in practice. Hence, how to generate a smooth path is an important task after we obtain the main mid-points.

Some assumptions are given and used in the sequel.

Assumption 1: The obstacles in mobile robot motion environment are represented by circles and there are only static obstacles in the map.

Assumption 2: The kinematic constraints of the mobile robots are not taken into consideration. It is assumed that the mobile robots have a certain size, and the radius of obstacle is expanded according to the size of robots.

3. Improved Hybrid GWO-WOA Method

In this section, an improved hybrid GWO-WOA method with tent chaotic strategy is proposed. The schematic of the proposed hybrid GWO-WOA method is depicted as in Fig. 1.

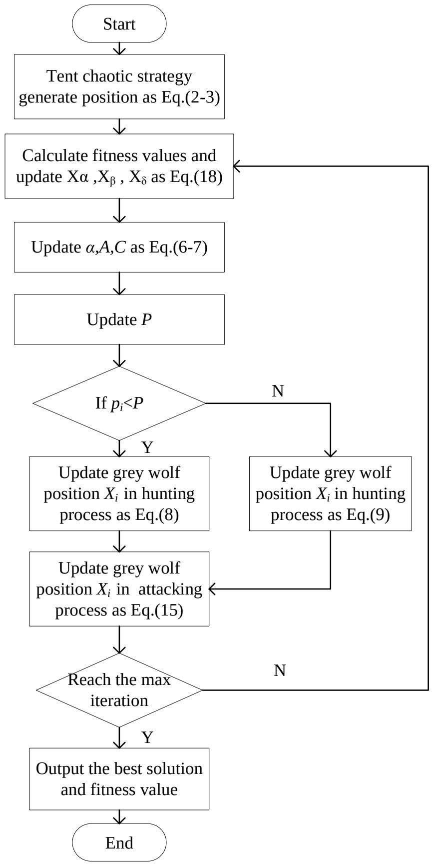

Fig. 1. The schematic of the proposed algorithm

According to [12], the hierarchy of grey wolves ranks as α, β, δ and ω, in which α is the leader, which is responsible for making decisions of hunting and other important issues. Based on the grey wolves hierarchy, the tasks of group hunting are described as: social hierarchy, encircling prey, hunting, attacking prey and search for prey. In this paper, the proposed method is designed by improved these steps.

3.1. Tent chaotic initialization strategy

In traditional GWO, the initial positions of grey wolves are generated randomly. However, although the randomness of the initial positions is ensured, the initial position of some individuals is still far away from the optimal point, such that the convergence rate and solution accuracy will be reduced. In [13], the tent chaos strategy is introduced to initialize the population. The structure of tent mapping is simple and it has great ergodicity and effectively solves the defect of initialization method. It is possible to make grey wolf individuals traverse all positions of the search space as far as possible.

The tent chaotic mapping function is described as:

$$r_{i,j} = \begin{cases} r_{i,j}/q, & 0 < r_{i,j} < q \\ (1-r_{i,j})/q, & q \leq r_{i,j} \leq 1 \end{cases} \quad \ldots\ldots \quad (2)$$

where $r_{i,j}$ is the jth of the function value from the ith grey wolf to the jth mapping. q is a given tent coefficient.

Then with the help of mapping $r_{i,j}$, the grey wolf population is generated as:

$$x_{i,j} = r_{i,j} * (u_b - l_b) + l_b \quad \ldots\ldots\ldots \quad (3)$$

where u_b and l_b are the given upper and lower bound of the solution space, respectively. $x_{i,j}$ is the jth dimension of the ith grey wolf.

3.2. Encircling prey

Grey wolves encircle prey during hunting. In order to mathematically model the encircling behaviour, the following equations are proposed:

$$\vec{D} = |\vec{C} * \vec{X}_p(t) - \vec{X}(t)| \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} * \vec{D} \quad \ldots\ldots\ldots \quad (5)$$

where t indicates the current iteration, $\vec{A}$ and $\vec{C}$ are coefficient vectors, $\vec{X}_p$ is the position vector of the prey, $\vec{X}$ indicates the position vector of a grey wolf.

The vectors $\vec{A}$ and $\vec{C}$ are calculated as follows:

$$\vec{A} = 2 * \vec{\alpha} * \vec{r}_1 - \vec{\alpha} \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$\vec{C} = 2 + \vec{r}_2 \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

where components of $\vec{\alpha}$ are linearly decreased from 2 to 0 over the course of iterations and $\vec{r}_1$, $\vec{r}_2$ are random vectors in [0, 1].

3.3. Hunting

In order to improve convergence performance of the grey algorithm, we hybridize WOA[14] with GWO algorithm[15]. WOA uses logarithmic spiral problems so it covers broader areas in uncertain search spaces[16]. In order to mathematically model hunting behaviour, it is assumed that there are two kinds of behaviour to position. The following spiral and hunting position update equations are developed based on different probability of each behavior as follows:

$$\vec{X}_\alpha(t+1) = \begin{cases} \vec{X}_\alpha(t) - \vec{A} * \vec{D}_\alpha, & p_1 < P \\ \vec{D}_\alpha * e^{b*l} * \cos(2*\pi*l) + \vec{X}_\alpha(t), & p_1 \geq P \end{cases} \quad (8)$$

$$\vec{X}_\beta(t+1) = \begin{cases} \vec{X}_\beta(t) - \vec{A} * \vec{D}_\beta, & p_2 < P \\ \vec{D}_\beta * e^{b*l} * \cos(2*\pi*l) + \vec{X}_\beta(t), & p_2 \geq P \end{cases} \quad (9)$$

$$\vec{X}_\delta(t+1) = \begin{cases} \vec{X}_\delta(t) - \vec{A} * \vec{D}_\delta, & p_3 < P \\ \vec{D}_\delta * e^{b*l} * \cos(2*\pi*l) + \vec{X}_\delta(t), & p_3 \geq P \end{cases} \quad (10)$$

$$\vec{X}(t+1) = \frac{\vec{X}_\alpha(t+1) + \vec{X}_\beta(t+1) + \vec{X}_\delta(t+1)}{3} \quad (11)$$

where e is natural constant, b is a constant for defining the shape of the logarithmic spiral, $\vec{D}_\alpha$, $\vec{D}_\beta$, $\vec{D}_\delta$ respectively indicate the distance of the ith wolf to the prey. $\vec{X}_\alpha, \vec{X}_\beta, \vec{X}_\delta$ indicate α, γ, δ position respectively. b is a constant for defining the shape of the logarithmic spiral. l is a random number $l \in [0,1]$.

3.4. Search for prey and attacking prey

In order to obtain the mathematical model of approaching prey, the value of $\vec{\alpha}$ is decreased. Note that the fluctuation range of $\vec{A}$ is also decreased by $\vec{\alpha}$. In other words $\vec{A}$ is a random vector about the $\vec{\alpha}$, where $\vec{\alpha}$ is decreased from 2 to 0 over the course of iterations. When random values of $\vec{A}$ are in [-1, 1], the next position of a search agent can be in any position between its current position and the position of the prey.

With the operators proposed so far, the GWO algorithm is easily to stagnation in local solutions with these operators. In this paper, strategy is replaced by the whale algorithm mechanism. When $|\vec{A}| \geq 1$, it is forced ω wolves to randomly select other members of the population to update their position, rather than the current best.

$$\vec{D} = |\vec{C} * \vec{X}_{rand} - \vec{X}(t)| \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A} * \vec{D} \quad \ldots\ldots\ldots \quad (13)$$

where $\vec{X}_{rand}$ is a randomly selected individual of grey wolf populations.

When $|\vec{A}| < 1$, ω wolves rely on the best wolf to update position.

4. GWO-WOA-based Path Planning

In this section, a smooth path planning method is proposed based on the proposed hybrid GWO-WOA method, which considers the collision-free and smoothness performances simultaneously.

4.1. Two-dimensional Search method

Considering that there are D points in a path, since start point and goal point are known, the problem of path planning using GWO-WOA method is to find D-2 points in the global space, which makes the path connected by adjacent points does not pass through obstacles, and the length of the path from the start point to the end point is the shortest, it is an optimization problem. The position of each wolf in population is described as a vector with D-2 dimensional, which is represented as:

$$\eta = \begin{pmatrix} x_1 & \cdots & x_i & x_{i+1} & \cdots & x_{D-1} \\ y_1 & \cdots & y_i & y_{i+1} & \cdots & y_{D-1} \end{pmatrix} \quad \ldots \quad (14)$$

where x_i, y_i $(i = 1, \cdots, D-1)$ are both unknown that need to be calculated.

In order to make the path smooth, this paper take the interpolation and spline[17] technique into consideration by interpolating h points between point (x_{i-1}, y_{i-1}) and point (x_i, y_i), hence, these points can be expressed as:

$$\begin{cases} \tilde{X}_i = \underbrace{x_{i-1} + x_{i1} + \cdots + x_{ih} + x_i}_{h+2} \\ \tilde{Y}_i = \underbrace{y_{i-1} + y_{i1} + \cdots + y_{ih} + y_i}_{h+2} \end{cases} \quad . \; . \; . \; . \; . \; . \; . \; (15)$$

This two-dimension vector indicates a path node. All the selected nodes could be connected one by one as the step going on until getting the target, so these nodes can indicate a path.

In this paper, the obstacles are assumed to be circles, in order to deal with the issue of collision avoidance, the radius of corresponding jth obstacle circle is r_j, which is shown as in Fig.2. The distance of the line between the path nodes and the center of the obstacle circle is defined as dis_{obs} and formulated as:

$$dis_{obs} = \sqrt{(x_{i,j} - x_{obs})^2 + (y_{i,j} - y_{obs})^2} \quad . \; . \; . \; (16)$$

where $x_{i,j}$ and $y_{i,j}$ are the horizontal and vertical coordinates of the path node after interpolate. (x_{obs}, y_{obs}) is the center of an obstacles.

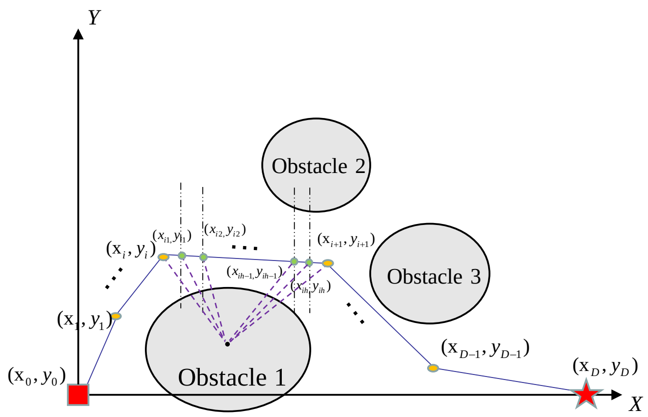

Fig. 2. The schematic of path planing with two dimensional search

If it holds that

$$r_j \leq dis_{obs} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (17)$$

then, the path is a collision-free path, otherwise the path intersects the obstacle. The fitness value of each search agent is calculated as:

$$dis = \sum_{i=1}^{D} \sum_{j=1}^{h} \sqrt{\left(x_{i-1,j-1} - x_{i,j}\right)^2 + \left(y_{i-1,j-1} - y_{i,j}\right)^2} \; (18)$$

where $(x_{i-1,j-1},\, y_{i-1,j-1})$, $(x_{i,j},\, y_i)$ are the two coordinate points of the ith path.

Overall, the process of multi-objective path planning under two-dimensional search mechanism can be concluded as the following steps:

Step 1: Initialize the grey wolf population X_i based on ten chaos strategy by Eq.(3).

Step 2: Smooth the path based on spline interpolation technique by Eq.(15).

Step 3: Determine whether the path is collision-free by Eq. (17).

If the path is collision-free, the fitness value is kept, otherwise, the fitness is replaced by a very big value Υ.

Step 4: Calculate the fitness values of these point by Eq.(18).

Step 5: If the max iteration reaches, the best fitness value and the best solution are output. Otherwise, return to Step 2.

Although this search method can solve the path planning problem. However it maybe time-consuming. In order to deal with the problem of long optimization time caused by large amount of data, in the next subsection, a new one-dimensional search method is proposed.

4.2. One-dimensional search method

In order to reduce the complexity of the algorithm and reduce the calculation time, we focus on the one dimensional search technique, that is, making the values of X axis but only optimize the values of Y axis with the conversion of coordinate model.

The position of each wolf is represented by a one-dimension vector, the position of wolf η expressed as

$$\eta = \begin{pmatrix} y_1 & \cdots & y_i & y_{i+1} & \cdots & y_{D-1} \end{pmatrix} \quad . \; . \; . \; . \; (19)$$

where y_i $(i = 1, \cdots, D-1)$ indicates Y axis coordinates on the path.

Based on [18], in this paper, we smooth the path based on spline interpolation technique before obstacle detection on one dimensional search. So the path is more precise and smooth.

Step 1: Transform the original coordinate system into a new coordinate whose X axis is the connection line from the starting point and ending point.

$$\theta = arctan \frac{x_D - x_0}{y_D - y_0} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (20)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} \quad . \; . \; (21)$$

where θ is the rotation angel of the coordinate system, the point (x, y) is in original coordinate, the point (x', y') is in the new coordinate.

Step 2: The segment from start point to end point is divided into D equal portions, as shown in Fig.3.

Since the new X' axis is fixed and these points of horizontal axis is equidistant ($X = \{x_0, x_1, x_2, x_3 ..., x_D\}$), only the vertical coordinate Y' of each node on the vertical lines need to be optimized, that is the unknown vector is $Y = \{y_1, y_2, y_3 ..., y_{D-1}\}$. In this way, two dimensional path search is reduced to one dimensional search. The path planning problem is transformed into a $D-2$ dimensional function optimization problem.

Step 3: Smooth the path based on spline interpolation technique. In order to reduce the optimization dimension of the problem and smooth the line curve, the path curve is constructed by using spline interpolation method under the premise of ensuring the solving accuracy. To be

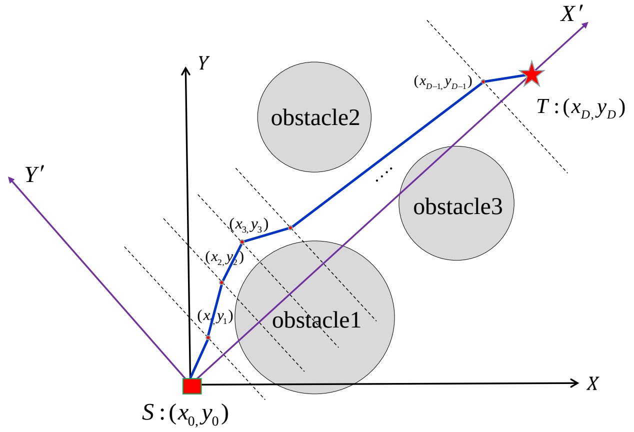

Fig. 3. The schematic of path planing with one dimensional search

specific, divide the X' and Y' coordinates of the adjacent nodes of the path into n points and create a spline as Eq.(15).

Step 4: Correspondingly, the fitness function of each wolf is selected as Eq.(18).

Calculate each wolf's fitness value, if the path is in contact with an obstacle, give a penalty value.

Step 5: Inverse conversion the coordinates to original in final optimal path.

The point values of the path are transformed into the original coordinate as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{22}$$

The proposed hybrid GWO-WOA method for mobile robot path planning is shown in Algorithm 1.

5. Simulation Results

In this section, the proposed hybrid GWO-WOA method is demonstrated by benchmarked functions and robot path planning problem, respectively.

5.1. Bench mark functions comparison

In this subsection, the proposed method is benchmarked on 5 functions which are listed in Table 1, where Range is the boundary of the functions' search space, and f_{min} is the corresponding minimum value.

Table 1. Benchmark functions

	Function	Range	f_{min}
F1	$\sum_{i=1}^{N} x_i^2$	[-100,100]	0
F2	$\sum_{k=1}^{N} \lvert x_i^2 \rvert + \prod_{i=1}^{N} \lvert x_i \rvert$	[-10,10]	0
F3	$max x_i\{\lvert x_i \rvert, 1 \leq i \leq n\}$	[-100,100]	0
F4	$\sum_{i=1}^{10} \left[(X-a_i)(X-a_i)^T + c_i \right]^{-1}$	[0,10]	-10.5363
F5	$\sum_{i=1}^{n-1} \left[100\left(x_{i+1}-x_i^2\right)^2 + (x_i-1)^2 \right]$	[-30,30]	0

Algorithm 1 Hybrid GWO algorithm for one-dimensional path planning

```
The grey wolf population is initialized by tent chaos strategy based on Eq.(2-3)
Given α, then A, C are calculated by Eq.(6–7)
Transform the original coordinate by Eq.(20–21)
Interpolate and spline X_i by Eq.(19)
for all X_i do
    Evaluate fitness by Eq.(18)
end for
Get the first three best wolves positions X_α, X_β, X_δ.
while t ≤ N_max (Maximum Iterations) do
    for all X_i do
        Update A⃗, C⃗, l and p
        if p ≤ 0.5 then
            if |A⃗| ≥ 1 then
                Update the position of the current search member by Eq.(4-5)
            else
                Select a random search member X⃗_rand
                Update the position of the current search member by Eq.(12-13)
            end if
        else
            Update the position of the present search member by using Eq.(8-11)
        end if
    end for
    Evaluate fitness for all search members by Eq.(18)
    Update X_α, X_β, X_δ
    t = t + 1
end while
Return X_α and X_α of fitness
Inversely transform the coordinates in final optimal path into the original coordinate by Eq.(22)
Output the value of X_α and fitness of X_α
```

In order to verify the superiority of the proposed method, it is compared with the GWO, PSO, WOA. For each benchmark function, the above algorithm was run 10 times starting from same populations randomly generated and the dimension of the function is 30. Statistical results, the optimization solutions are obtained as in Table 2. Besides, the iteration process are depicted as in Fig.4-8, the convergence rate of GWO-WOA in Figs.4-8 is better than the other algorithms. From the results, we can see that the proposed GWO-WOA method is able to provide competitive results on the benchmark functions.

5.2. Planing path comparison

In this subsection, the proposed method is demonstrated by a path planning problem. In this section, GWO-WOA is benchmarked using two simulation cases in table. it is also compared with GWO, PSO, WOA. It is noteworthy that the above all algorithm used interpolation and spline technique before obstacle detection on one dimensional search.

In the first case, the starting point is set as (20,80) and

Table 2. Simulation results of Algorithms

	GWO	PSO	WOA	**GWO-WOA**
F1	1.2488e-62	4.7799e-10	3.1636e-108	**2.5591e-102**
F2	1.783e-35	4.2067e-05	1.1003e-60	**2.284e-70**
F3	4.4661e-17	0.30541	0.92221	**5.494e-21**
F4	-10.5269	-10.5364	-10.5331	**-10.5364**
F5	0.002521	0.025413	0.033233	**0.0015186**

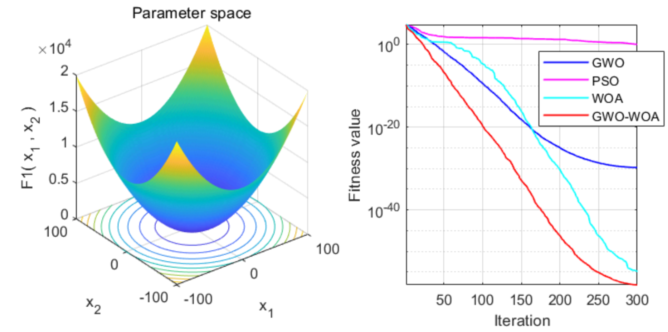

Fig. 4. Iteration results of Function 1

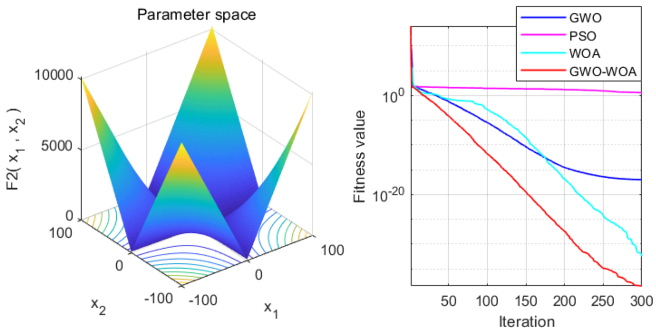

Fig. 5. Iteration results of Function 2

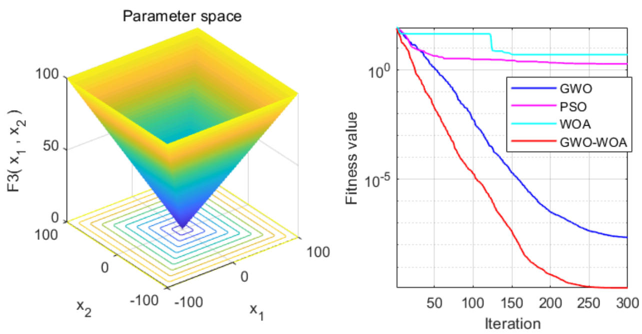

Fig. 6. Iteration results of Function 3

the ending point is set as (80,20) in Fig.9. In the second case, the starting point is (0,0) and the ending point is (100,100) in Fig.11. We define three nodes on the path, search agents are 300 search points between two adjacent points and the iteration is 300 times.

The experimental results are listed in Tables 3-4. The

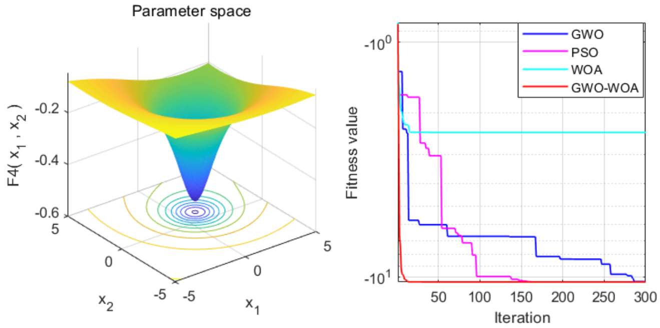

Fig. 7. Iteration results of Function 4

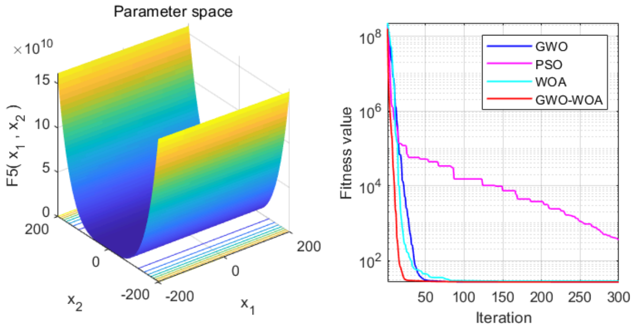

Fig. 8. Iteration results of Function 5

results are averaged over 10 independent runs and the Best, Worst, Average represent the optimal fitness value, worst fitness value, average fitness value.

Table 3-4 show the results for the two cases. From the results, we can see that one-dimension search time is shorter than two-dimensional time. In addition, whether it is one-dimensional or two-dimensional search, the results of these statistical variables prove that GWO-WOA has the best ability to avoid local minima. Both in the two cases, GWO-WOA are much smaller than the other algorithms in the Best, Worst, and Mean values of the results. Fig.10 and Fig.12 illustrate the convergence cures of all algorithms base on the two different Map of environment. As can be seen from these curves, the convergence rate of GWO-WOA in Fig.10 and Fig.12 is better than the other algorithms.

Table 3. Comparison results of path length in case 1

	Best	Worst	Average	Time
PSO-2	85.829	86.929	86.797	32.37s
WOA-2	85.725	86.725	86.436	31.42s
GWO-2	85.704	86.979	86.409	31.37s
GWO-WOA-2	**84.867**	**85.685**	**86.351**	**30.18s**
GWO-WOA-1	**85.685**	**86.161**	**85.514**	**25.85s**

Table 4. Comparison results of path length in case 2

	Best	Worst	Average	Time
PSO-2	142.46	152.56	147.55	31.49s
WOA-2	142.81	146.71	144.76	30.95s
GWO-2	142.54	148.41	145.48	32.48s
GWO-WOA-2	**142.34**	**146.31**	**144.33**	**30.19s**
GWO-WOA-1	**142.19**	**145.99**	**144.09**	**26.17s**

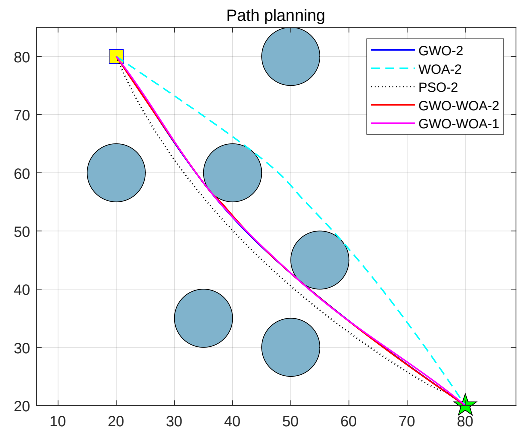

Fig. 9. Comparison results of path planning

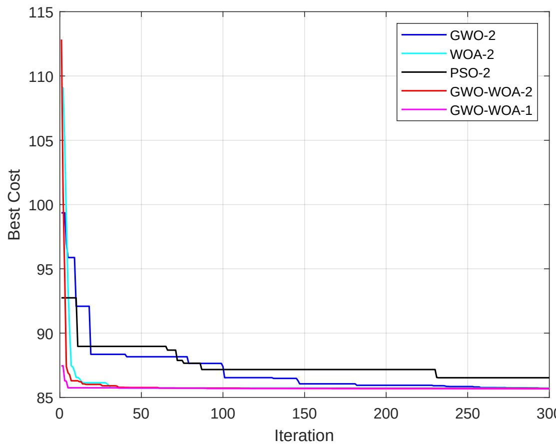

Fig. 10. Iteration results in case 1

6. Conclusion

This paper proposes an improved hybrid GWO-WOA method for collision-free path planning of mobile robots. First, the initialization of GWO is modified by the tent chaotic mechanism. Then the hybrid GWO-WOA method is designed by combining the advantages of the GWO and WOA. To obtain a smooth path, some important midpoints are calculated by the proposed hybrid GWO-WOA method and the others are generated by spline interpolation method. Besides, to reduce the computing burden, an one-dimensional search strategy is proposed. Finally, simulation results show that the hybrid algorithm has ad-

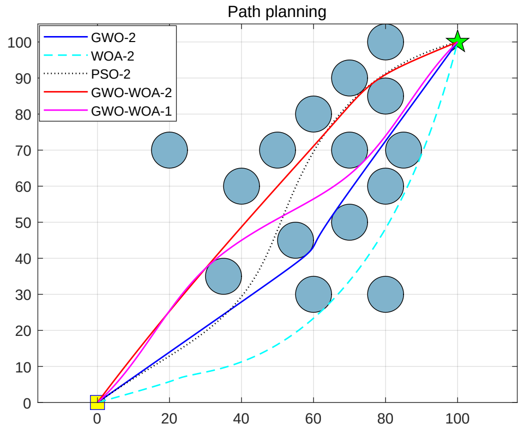

Fig. 11. Path planning results in case 2

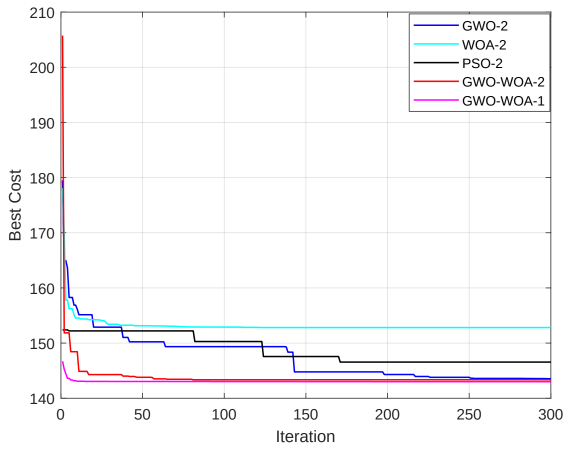

Fig. 12. Iteration results in case 2

vantages in the optimization and convergence speed.

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