Paper:

Stability Analysis of One-Step-Guess Estimator

Zhiyu Zhao,Hongbin Ma

Zhongguancun South Street, Haidian District, Beijing, No. 5
E-mail: wuhuaguocao@126.com
[Received 2021/08/01; accepted 2021/08/25]

Abstract.This paper focuses on an extremely simple adaptive method, one-step-guess(OSG) estimator, which has been ignored for a quite long time for its simple form. Previous researches have been concerned with the application of OSG in the field of adaptive control and adaptive tracking. Although the stability of the OSG has been discussed, a further investigation is necessary to analyze stability of OSG. The contribution of this paper aims at taking a deep look at the stability of OSG. The result shows that this simple estimator can be stable under mild conditions, and the error is bounded.

Keywords: One-step-guess, adaptive control, discrete-time, stability, nonlinear system.

1. Introdution

In field of automatic control, uncertainties and feedback are significant issues. In certain sense, the key to designing a proper control system is to design control laws to handle the former problems properly. Robust control and adaptive control are two basic methods to deal explicitly with uncertainty in control systems. Comparing with robust control, adaptive control is more attractive and challenging due to its nature. It involves some ideas of learning in a certain sense so as to be able to deal with larger uncertainties, with the cost of encountering some difficulties of complex nonlinear closed loop system analysis.

However, the control law of adaptive control usually involves a highly nonlinear closed-loop system of adaptive controller, which leads to a heavy computation burden. Furthermore, current adaptive control laws focus on the continuous-time system[1–3], but in modern control systems, more and more controllers have to be implemented on todays embedded hardware. So the development of adaptive control method with small computing burden to deal with large uncertainties for discrete system is necessary.

In order to deal with the problem mentioned above, a class of the scalar discrete-time adaptive control system based on an extremely simple one-step-guess (OSG) estimator is put forward. The basis idea of OSG adaptive controller is an intuitive idea to estimate the unknown parameter with only the information available from one step, and its form is much simpler than and different from that of other widely used adaptive controllers based on least-square algorithm or gradient-descent algorithms, which suggests that this method have less computation burden compared other adaptive control method.

The OSG controller is also introduced into the scenarios of multi-agent systems. Decentralized adaptive tracking problem for a discrete-time multi-agent system is investigated in [8]. A decentralized adaptive tracking with One-Step-Guess estimator is proposed. It has advantage over least-squares algorithm and gradient-type algorithms on the computational complexity.

OSG adaptive method is indeed governed by a time varying nonlinear difference equation, which was not thoroughly investigated in previous studies of difference equations. There have been researchers concerned with the nonlinear difference equations. They investigate the global attractivity of the recursive sequences $x_{n+1} = -\alpha \frac{x_{n-1}}{\beta \pm x_n}$ under specified conditions [4]. The recursive sequences $y_{n+1} = A + \frac{y_n}{y_{n-k}}$ [5].

Among all these results, the coefficients concerned are time-invariable. Especially, the OSG estimator focuses on the time-variable parameters, which generates a more complicated error analysis and stability analysis. The previous researches have cast light on the stability of the OSG estimator, but the criterion is complex and unpractical[6, 7].

After a deep look at the OSG estimator, a new stability criterion is put forward, which shows that the stability of the algorithm can be guaranteed only by ensuring that the error bounds at the initial time meet the specific requirements.

This contribution is organized as follows. The OSG estimator setting and algorithm are introduced in Section 2. The bound of errors caused by noise are analyzed in Section 3, where the upper bound is derived. The new stability criterion is in the section 4.This is then verified by numerical simulations in Section 5. Section 6 concludes this article.

2. The OSG estimator setting and algorithm

2.1. Problem statement

The OSG estimator consider the scalar discrete-time system

$$x(k+1) = ax(k) + u(k) + w(k) \tag{1}$$

where $x(k)$ and $u(k)$ denote the system output and input, respectively; a is an unknown plant parameter; and $w(k)$ is the system noise. Here $x(k)$ is supposed to be available for measurement. The initial value of system state is an arbitrary real number, $x(0) = x_0$.

2.2. Design of One-Step-Guess (OSG) Estimator

The one-step-guess of a, which only utilize the previous one step information, is given as following:

$$\hat{a}(k) = \frac{x(k) - u(k-1)}{x(k-1)} \tag{2}$$

It is notable that it is easy to obtain the accurate estimation of the system parameter by one-step-guess estimator for nominal system, which means that the system output can track the reference signal without error.

In order to get the desired state $x^*(k)$, the input is set to be

$$u(k) = x^*(k+1) - \hat{a}(k)x(k) \tag{3}$$

Then an useful expression to estimate a is gotten[7]

$$\hat{a}(k) = \frac{x(k) - x^*(k)}{x(k-1)} + \hat{a}(k-1) \tag{4}$$

The form of OSG mentioned before is often called primordial OSG adaptive method. It is notable that the value of x(k-1) maybe zero due to the effect of noise in the primordial OSG adaptive method, which leads to the undesirable zero division.

This problem can be solved by the regularization, the update law of $\hat{a}$ is

$$\hat{a}(k) = \frac{x(k-1)(x(k) - u(k-1))}{x^2(k-1) + \lambda} \tag{5}$$

in which λ is a small positive real number. As $x(k-1)^2 + \lambda \geq \lambda > 0$, the zero division problem is solved. This method is called parameter regularized OSG.

3. Error Analysis

The error is defined as follows.

$$e(k) = x(k) - x^*(k) \tag{6}$$

in which $x^*(k)$ represents the desired state while $x(k)$ represents the state we get at the instant k.

Using 1 and 4, we can get that

$$e(k) = -\frac{w(k-2)}{x(k-2)}x(k-1) + w(k-1) \tag{7}$$

Investigate the absolute value of $e(k)$,

$$|e(k)| \leq |\frac{|w(k-2)|}{|x(k-2)|}|x(k-1)| + |w(k-1)| \tag{8}$$

which implies that the error $e(k)$ satisfies the recursive sequence correlated to the noise and states. Assume that the noise is white noise with variance of v and mean value of 0.

$$\mathbb{E}(e(k)) = 0 \tag{9}$$

The variance of $e(k)$ is

$$\begin{aligned} \mathbb{E}(e^2(k)) \leq & (\frac{x^2(k-1)}{x^2(k-2)} + 1)v \\ = & (\frac{(x^*(k-1) + e(k-1))^2}{(x^*(k-2) + e(k-2))^2} + 1)v \end{aligned} \tag{10}$$

Similarly, the error of parameter regularized OSG is

$$\begin{aligned} e(k) = & \frac{\lambda a + x(k-2)(a(k-2) - x(k-1) + u(k-2))}{x(k-2)^2 + \lambda} \\ & + w(k-1) \\ = & \frac{\lambda a - x(k-1)w(k-2)}{x^2(k-2) + \lambda} + w(k-1) \end{aligned} \tag{11}$$

4. Stability Criterion

Before introducing the main theorem, we give the assumption that the problem satisfies.

Assumption 4.1: The sequence of $x^*(k)$ is bounded,$b_1 \leq |x^*(k)| \leq b_2$ and the amplitude of noise $b \leq \frac{(\beta-1)b_1 - b_2}{\beta^2}$,$\beta > max\{|x^*(1) - x(1)|/b, |x^*(0) - x(0)|/b\}$

Theorem 4.1: If the noise is bounded, $|w(k)| < b$,$x^*(k)$ and $w(k)$ satisfy 4.1,$\forall k \geq 0$, the error of primordial OSG adaptive method $e(k)$ is bounded.

Proof:

If there exist $|e(k-1)| \leq \eta, |e(k)| \leq \eta$,

$$\begin{aligned} |e(k+1)| = & |-\frac{w(k)}{x(k-1)}x(k) + w(k+1)| \\ \leq & \frac{|w(k)|}{|x(k-1)|}|x(k)| + |w(k+1)| \\ \leq & \frac{b_2 + \eta}{b_1 - \eta}b + b = \frac{b_1 + b_2}{b_1 - \beta b}b \leq \eta \end{aligned}$$

Similarly, we can obtain that $|e_{t+n}| < \eta, n > t$. As $|e(0)| < \beta b, |e(1)| < \beta b, |e(n)| < \beta b$. Let $\eta = \beta b$, the tracking error $e(k) = x(k) - x^*(k)$ is bounded. Using 10, the variance of $e(k)$ is

$$\mathbb{E}(e(k)) \leq ((\frac{b_2 + \beta b}{b_1 - \beta b})^2 + 1)v \leq (\beta - 1)^2 v \tag{12}$$

■

Theorem 4.2: If the desired state sequence $x^*(k)$ and noise $w(k)$ is bounded, $b_1 \le x^*(k) \le b_2, |w(k)| < b, \forall k \ge 0$. for parameter regularized OSG, the conclusion is $\sup_{k\to\infty} e(k) < \infty$

Proof: From the condition, the $e(0)$ and $e(1)$ are bounded.

Case 1. Assume that $e(k-1)$, $e(k-2)$ is bounded,$e(k-1), e(k-2) < \eta$. Using 11,

$$\begin{aligned} |e(k)| &\le \frac{|\lambda a| + |x(k-1)||w(k-2)|}{x^2(k-2)+\lambda} + |w(k-1)| \\ &< a + \frac{b|\eta + b_2|}{\lambda} + b < \infty \end{aligned} \tag{13}$$

which means $e(k)$ is bounded. So $e(k+1)$ is also bounded. $e(j)$ is bounded, $\forall j > k+1$.

Case 2. Assume that $e(k-1)$ is bounded while $e(k-2)$ is not bounded.Then $x(k-2) = e(k-2) + x^*(k-2)$ is not bounded. Using 11,

$$|e(k)| \le \frac{|\lambda a| + |x(k-1)||w(k-2)|}{x^2(k-2)+\lambda} + |w(k-1)| \to 0 \tag{14}$$

So $e(k-1)$ and $e(k)$ are bounded. From case 1, $e(k+1)$ is bounded.$e(j)$ is bounded, $\forall j > k+1$.

Case 3. Assume that $e(k-2)$ is bounded while $e(k-1)$ is not bounded. Using 11, $e(k)$ is not bounded.

$$\left|\frac{e(k)}{e(k-1)}\right| = \left|\frac{w(k-2)}{x^2(k-2)+\lambda}\right| < \infty \tag{15}$$

So

$$|e(k+1)| = \left|\frac{\lambda a - x(k-1)w(k-2)}{x^2(k-2)+\lambda} + w(k-1)\right| \to 0 \tag{16}$$

From case 2, $e(j)$ is bounded, $\forall j > k+1$.

Case 4. Assume that both of $e(k-2)$, $e(k-1)$ are not bounded. From case 3, $e(j)$ is bounded, $\forall j > k$.

To summarize, $\exists k < \infty$, $e(j)$ is bounded, $\forall j > k$. ■

5. Numerical Simulations

5.1. Primordial OSG Adaptive Method

In the simulation experiment, $a = 10$, $x^*(k)$ are samples from a uniform distribution with lower bound $b_1 = 10$, upper bound b_2=10.1, The absolute value of noise has upper bounded $b = 1$.

The Fig.1 shows that the estimation error of primordial OSG adaptive method with the iteration times. In the Fig.2, the orange dots shows the desired states, while the blue dots shows the actual states.

The result in Fig.1 shows that the maximum appears in the first two estimates, which is consistent with the conclusion of 4.1. Besides, the maximum of error after the second observation is 0.927672366794237, which is smaller than the bound of noise.

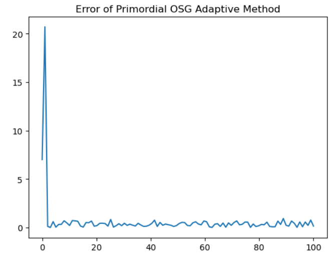

Fig. 1. Error of Primordial OSG Adaptive Method

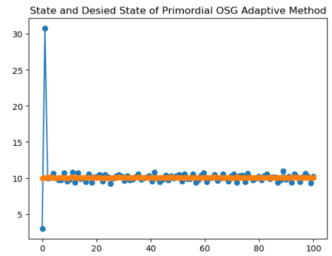

Fig. 2. State and Desied State of Primordial OSG Adaptive Method

5.2. Parameter Regularized OSG

In the simulation experiment, $a = 10$, $x^*(k)$ are samples from a uniform distribution with lower bound $b_1 = 10$, upper bound b_2=20, The absolute value of noise has upper bounded $b = 1.\lambda = 0.01$

The Fig.3 shows that the estimation error of parameter regularized OSG adaptive method with the iteration times. In the Fig.4, the orange dots shows the desired states, while the blue dots shows the actual states.

The result in 3 also shows that the maximum appears in the first two estimates, which is consistent with the conclusion of 4.1. Besides, the maximum of error after the second observation is 0.9207555558660623, which is smaller than the bound of noise.

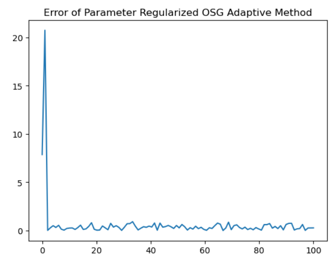

Fig. 3. Error of Parameter Regularized OSG Adaptive Method

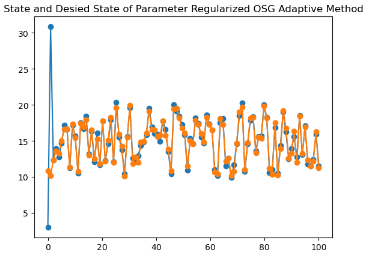

Fig. 4. State and Desied State of Parameter Regularized OSG Adaptive Method

6. Conclusion

As shown in the Fig 1, the max of error is $e(1)$, and the rest of error $e(k) \leq e(1)$, which supports the conclusion of Theorem 4.1. Fig3 shows that the max of error is $e(1)$, and the rest of error $e(k) \leq e(1)$, which supports the conclusion of Theorem 4.2.

References:

[1] Chien and Fu,An adaptive variable structure control for a class of nonlinear systems,Systems & Control Letters.

[2] Wang, L. X., Adaptive fuzzy systems and control : design and stability analysis.

[3] Ushida, S. and Kimura, H., Adaptive control of nonlinear system with time delay based on the feedback error learning method. Sice Sice Conference. IEEE.

[4] El-Owaidy, H. M. and Ahmed, A. M. and Mousa, M. S., On the recursive sequences $x_{n+1} = -\alpha \frac{x_{n-1}}{\beta \pm x_n}$, applied mathematics & computation.

[5] R.M and Abu-Saris and and and R and DeVault, Global stability of $y_{n+1} = A + \frac{y_n}{y_{n-k}}$, Applied Mathematics Letters.

[6] Rong, L. and Ma, H. and Wang, M.,Nontrivial closed-loop analysis for an extremely simple one-step-guess adaptive controller,Proceedings of the 2011 Chinese Control and Decision ConferenceCCDC

[7] Ma, H. and Rong, L. and Wang, M. and Fu, M.,Adaptive tracking with one-step-guess estimator and its variants,Control Conference (CCC), 2011 30th Chinese

[8] Rong L , Ma H , Yang C , et al. Decentralized adaptive tracking with one-step-guess estimator[C]// 9th IEEE International Conference on Control and Automation, ICCA 2011, Santiago, Chile, December 19-21, 2011. IEEE, 2011.